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MOMENT-STRAIN RELATIONSHIPS IN ELASTIC-PLASTIC BENDING OF BEAMS

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Theoretical expressions for bending moment versus depth of elastic-plastic interface for both rectangular, tubular, and solid circular beams are developed. Theoretical expressions for strain as a function of geometry and depth of elastic-plastic interface for each type of cross section are derived. The possibility of a strain singularity occurring as the elastic-plastic interface approaches the neutral axis is pointed out. The assumption that "planes remain

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plane" is examined. Experimental data is presented which supports the assumption. The basis for determining residual or permanent strains and expressions for their calculation are given. Finally a brief section of strain-curvature relationships is presented.

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INTRODUCTION

This report is one of a series dealing with the theoretical and experimental aspects of elastic-plastic bending of beams. For the most part the material presented in this report is not new and can be found in the literature although in quite fragmented form. Therefore it was felt there was a need to put the material into a more organized form which hopefully would be more meaningful to those engaged in development or processing type activities. The writer has gone into a fair amount of detail in developing the theoretical expressions on a step-by-step basis. This should aid the reader in tying this material to that obtained in undergraduate strength of material type courses. Although the theory is rather simple, it can give a certain amount of insight into the complex area of gun tube straightening.

DEVELOPMENT OF BASIC EQUATIONS

Bending Moment - Depth of Elastic-Plastic Interface

As a simple example to illustrate the approach in calculating the bending moment, we can consider the case of a rectangular beam. The moment is determined from two integrals. We integrate through the first region where the stress σ_V varies linearly with distance from the neutral axis (NA). The first integral for the partial moment which we shall call M_1 is then:

$$M_1 = \int_o^m \sigma_V(bdy)y \qquad (1)$$

As shown in Figure 1, ρ is the distance from the upper fiber to the elastic-plastic interface, m is the distance from the NA to the elastic-plastic interface, and y is the distance from the NA to the differential element. By ratio we can determine that

$$\sigma_v = \sigma_y \left(\frac{y}{m}\right) \tag{2}$$

Substituting this expression into equation (1) above, we obtain

$$M_1 = \int_o^m \sigma_y\left(\frac{y}{m}\right)(bdy)y \tag{3}$$

Now performing the simple integration, we obtain the expression for M_1.

$$M_1 = \frac{\sigma_y b}{3} m^2 \tag{4}$$

The second integral for the partial moment which we shall call M_2 is evaluated from m to h/2. Since strain hardening is neglected for the simple case being studied, the stress σ_v contained in equation (1) above is constant and equal to σ_y. The second integral is then:

$$M_2 = \int_m^{h/2} \sigma_y(bdy)y \tag{5}$$

Evaluating this simple integral, we obtain the following expression for M_2

$$M_2 = \frac{\sigma_y b}{2} \left[\frac{h^2}{4} - m^2\right] \tag{6}$$

Combining expressions (4) and (6) for the partial moments M_1 and M_2 we obtain the total moment in terms of the geometry of the cross section, the yield stress, and the depth of the elastic-plastic interface.

$$M = \frac{\sigma_y b}{6}\left[h^2 + 2\rho h - 2\rho^2\right] \qquad (7)$$

Here we have made use of the relationship $m = (h/2) - \rho$ and a factor of 2 has been inserted to take care of the bottom half of the cross section. Equation (7) can be easily programmed even using a hand calculator to give values as a function of the various parameters. Figure 2 shows a family of curves from this expression for different values of the yield stress σ_y. Two special cases can be derived from equation (7). If $\rho = 0$, $M = M_y$ (the yield moment) and we have

$$M_y = \frac{\sigma_y b h^2}{6} \qquad (8)$$

If $\rho = h/2$, the elastic-plastic interface coincides with the NA. The fully plastic moment then becomes

$$M_{fp} = \frac{\sigma_y b h^2}{4} \qquad (9)$$

The ratio of the fully plastic moment to the yield moment is a constant for rectangular beams.

$$M_{fp}/M_y = 1.5 \qquad (10)$$

Or we could say that the fully plastic moment is 50 percent greater than the yield moment for a rectangular beam.

A second and somewhat more difficult example considers the case of a beam having a tubular cross section. To determine an expression for the bending moment we have to consider two cases. The first case is when the elastic-plastic interface lies between the outside surface and the bore. The second case is when the elastic-plastic interface is located between the NA and the bore. Case one will be discussed first. With the aid of Figure 3 we set up three equations as follows, using a similar approach as for the rectangular beam. The first integral accounts for the elastic stress acting on the area delimited by the large circle.

$$M_1 = 4\int_o^m \sigma_v \, y \, dA = 4\int_o^m \sigma_y \sqrt{R^2-y^2} \, dy$$

$$M_1 = 4\int_o^m \sigma_y \left(\frac{y}{m}\right) y \sqrt{R^2-y^2} \, dy$$

$$M_1 = \frac{4\sigma_y}{m} \int_o^m y^2 \sqrt{R^2-y^2} \, dy \tag{11}$$

Evaluating equation (11) from tables we obtain:

$$M_1 = \frac{4\sigma_y}{m} \left\{ -\frac{m}{4}(R^2-m^2)^{3/2} + \frac{R^2}{8}\left[m(R^2-m^2)^{1/2} + R^2 \sin^{-1}\left(\frac{m}{R}\right)\right]\right\} \tag{12}$$

where a factor 4 has been inserted to account for the double symmetry about the x and y axes.

The second integral subtracts the elastic stress for the small circle. This integral is the same as equation (11) above except r replaces R and the limits go from 0 to r. Hence,

$$M_2 = -\frac{4\sigma_y}{m} \int_o^r y^2 \sqrt{r^2-y^2} \, dy \tag{13}$$

Evaluating this gives:

$$M_2 = -\frac{\sigma_y}{2m} r^4 \sin^{-1}(1) = -\frac{\sigma_y \pi r^4}{4m} \tag{14}$$

Finally, to determine integral three, we integrate through the region from m to R where the stress σ_v takes on the constant value σ_y as follows:

$$M_3 = 4\int_m^R \sigma_v \, y \, dA = 4\int_m^R \sigma_y \, y \sqrt{R^2-y^2} \, dy$$

$$M_3 = 4\sigma_y \int_m^R y \sqrt{R^2-y^2} \, dy \tag{15}$$

Evaluating this we get:

$$M_3 = \frac{4\sigma_y}{3} (R^2-m^2)^{3/2} \tag{16}$$

Now combining M_1, M_2, and M_3 we obtain the desired expression for the total bending moment.

$$M = \frac{\sigma_y}{3} (R^2-m^2)^{3/2} + \frac{\sigma_y}{2} R^2 (R^2-m^2)^{1/2} - \frac{\sigma_y \pi r^4}{4m} + \frac{\sigma_y}{2} \frac{R^4}{m} \sin^{-1}\left(\frac{m}{R}\right) \tag{17}$$

Substituting the value for $m = R-\rho$, we obtain an expression for the bending moment in terms of yield stress, the geometrical parameters, and the depth of the elastic-plastic interface ρ for the region $r \leq m \leq R$.

$$M = \frac{\sigma_y}{3} (2R\rho-\rho^2)^{3/2} + \frac{\sigma_y}{2} R^2 (2R\rho-\rho^2)^{1/2} - \frac{\sigma_y \pi r^4}{4(R-\rho)} + \frac{\sigma_y}{2} \frac{R^4}{(R-\rho)} \sin^{-1}\left(\frac{R-\rho}{R}\right) \tag{18}$$

Specializing this equation for the case $\rho = 0$, we can obtain an expression for the yield moment.

$$M_y = \frac{\sigma_y \pi}{4R} (R^4-r^4) \tag{19}$$

We now set up the integrals for the second case where the elastic-plastic interface lies between the NA and the bore. This involves four equations as follows. Figure 4 is included as an aid to setting up these expressions.

$$M_1 = 4\int_o^m \sigma_y\left(\frac{y}{m}\right)y\sqrt{R^2-y^2}\ dy \qquad (20)$$

$$M_2 = -\ 4\int_o^m \sigma_y\left(\frac{y}{m}\right)y\sqrt{r^2-y^2}\ dy \qquad (21)$$

$$M_3 = 4\int_m^R \sigma_y\ y\sqrt{R^2-y^2}\ dy \qquad (22)$$

$$M_4 = -\ 4\int_m^r \sigma_y\ y\sqrt{r^2-y^2}\ dy \qquad (23)$$

Equation (20) accounts for the elastic stress acting on the large circle from 0 to m. Equation (21) subtracts the elastic stress acting on the small circle. Equation (22) accounts for the elastic-plastic stress acting on the large circle from m to R, while equation (23) subtracts the elastic-plastic stress acting on the small circle. Evaluating we obtain the following results:

$$M_1 = \frac{4\sigma_y}{m}\left\{-\frac{m}{4}(R^2-m^2)^{3/2} + \frac{R^2}{8}\left[m(R^2-m^2)^{1/2} + R^2\ \sin^{-1}\left(\frac{m}{R}\right)\right]\right\} \qquad (24)$$

$$M_2 = -\frac{4\sigma_y}{m}\left\{-\frac{m}{4}(r^2-m^2)^{3/2} + \frac{r^2}{8}\left[m(r^2-m^2) + r^2\ \sin^{-1}\left(\frac{m}{r}\right)\right]\right\} \qquad (25)$$

$$M_3 = \sigma_y(R^2-m^2)^{3/2} \qquad (26)$$

$$M_4 = -\frac{4}{3}\sigma_y(r^2-m^2)^{3/2} \qquad (27)$$

As before, the factor 4 accounts for the double symmetry. Specializing these results for the fully plastic condition $\rho = R$ gives $m = 0$. We then have

$$M_{fp} = \frac{4}{3} \sigma_y (R^3 - r^3) \tag{28}$$

Dividing equation (28) by equation (19) we have the ratio of the fully plastic moment to the yield moment

$$\frac{M_{fp}}{M_y} = \frac{16R}{3\pi} \frac{(R^3 - r^3)}{(R^4 - r^4)} \tag{29}$$

We see that for the tubular beam this ratio is not a constant as in the case of the rectangular beam, but depends on the radii of the cross section. The algebraic sum of the equations (24) to (27) gives the desired result for the bending moment in terms of the depth of the elastic-plastic interface parameter m in the region for $0 \leq m \leq r$. As before, these can be put in terms of the explicit parameter ρ by making the substitution $m = R - \rho$. A computer program for the solution of the equations belonging to the two different regions has been written and given the acronym MOMENTU. It contains two DO LOOPS -- one for the yield stress σ_y and the other for the depth of the elastic-plastic interface ρ. Results from the program are shown graphically in Figure 5 for the case where $R = 3.375$ inches and $r = 1.75$ inches and $\sigma_y =$ 162 Ksi and 172 Ksi.

Before leaving this section, we might extract additional information from the equations developed for tubular beams. By focusing on equations (24) through (27), we can derive a moment expression for a solid circular rod using equations (24) and (26) only. This is the same as neglecting to subtract the expressions in equations (25) and (27). Doing this we get

$$M = M_1 + M_3 = \frac{\sigma_y}{3} (R^2-m^2)^{3/2} + \frac{\sigma_y}{2} R^2(R^2-m^2)^{1/2} + \frac{\sigma_y R^4}{2m} \sin^{-1}\left(\frac{m}{R}\right) \qquad (30)$$

Now specializing this for the two cases as done before, we get values for the yield and fully plastic moments. Letting $m = R$, the condition for incipient yield, we get the following from the only non-zero third term.

$$M_y = \frac{\sigma_y \pi}{4} R^3 \qquad (31)$$

Letting $m = 0$, all three terms remain, but the third term becomes indeterminate requiring the use of l'Hospital's Rule. Differentiating and evaluating, the third term becomes equal to $\sigma_y R^3/2$ and the sum of the three terms gives the value for the fully plastic moment.

$$M_{fp} = \frac{4}{3} \sigma_y R^3 \qquad (32)$$

The moment ratio is then:

$$\frac{M_{fp}}{M_y} = \frac{\frac{4}{3} \sigma_y R^3}{\frac{\pi}{4} \sigma_y R^3} = \frac{16}{3\pi} = 1.6976 \cong 1.7 \qquad (33)$$

As in the case of the rectangular beam, the moment ratio is independent of the geometry for the solid circular rod. Of course the value of $16/3\pi$ could have been obtained more easily by specializing equation (29) for the case $r = 0$.

Strain-Curvature Relations

Figure 6 shows the relationship of the strains on the outside fiber of the beam to the yield strain. This is based on the assumption that "planes remain plane". While this assumption is rather easy to believe for the case of elastic bending, intuitively it is harder to accept when bending involves large plastic strains. McCullough[1] showed this assumption to be quite good in his experiments involving elastic-plastic bending of lead beams. Figures 7 and 8 show strain vs. depth of cross section for elastic strains as well as permanent strains after removal of the load. The strains were measured with resistance strain gages and are probably accurate to 3-4 percent. Based on McCullough's observations[1] and the data presented here, the assumption that "planes remain plane" appears to be valid. Based on ratio, we can then write

$$\varepsilon_{MAX} = \frac{h/2}{\frac{h}{2} - \rho}\, \varepsilon_y \tag{34}$$

as $\rho \to h/2$, the denominator approaches ∞ and we theoretically have a strain singularity. This expression can readily be adapted to circular tubes by replacing $h/2$ by the outside radius R. We then have

[1] McCullough, B. H., "An Experimental and Analytical Investigation of Creep in Bending," Trans ASME, Journal of Applied Mechanics, Vol. 55, 1935, p. 55.

$$\varepsilon_{MAX} = \frac{R}{R-\rho} \varepsilon_y \tag{35}$$

One can take advantage of the "planes remain plane" assumption to experimentally evaluate the large strains that sometimes occur on the outside fiber of the beam subjected to elastic-plastic bending. One can place gages at various depths or distances from the NA on a vertical line representing a given cross section of the beam. From a linear plot of strain vs. distance from the NA one can extrapolate to determine the strains at the extreme fibers.

From Hooke's law and the flexure formula from elementary bending theory, we can calculate a value for the psuedo elastic strain to be subtracted from the maximum elastic-plastic strain to determine the residual or permament strain.

$$\overset{*}{\varepsilon} = \frac{Mc}{EI} \tag{36}$$

Here M is the bending moment, c the distance from the NA to the outside fiber, E is the modulus, and I the moment of inertia. This of course assumes linear unloading. We then have:

$$\varepsilon_{res} = \varepsilon_{MAX} - \overset{*}{\varepsilon} \tag{37}$$

Finally to close out this small communication, we should include some remarks about curvature. From elementary calculus, the following expression can be obtained

$$\kappa = \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{3/2}} \tag{38}$$

For small deformations, the slope dy/dx is small compared to unity, hence the squared term is still smaller and can be neglected. So to a good approximation the curvature is equal to the second derivative of the deflection with respect to the distance along the beam. Then

$$\kappa \cong \frac{d^2y}{dx^2} \tag{39}$$

From the well-known Bernoulli-Euler formula we have:

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{40}$$

Hence

$$\kappa = \frac{M}{EI} \tag{41}$$

or

$$\frac{d^2y}{dx^2} \cong \kappa \tag{42}$$

It then appears that one method of determining the deflection is by integrating the curvature twice.

From elementary beam theory based on the assumption that "planes remain plane" we also have

$$\varepsilon = y/R_A \tag{43}$$

where ε is the strain on a fiber located at a distance y from the NA and R_A is the radius of curvature. Since $R_A = 1/\kappa$ we have a simple relationship between strain and curvature

$$\kappa = \frac{\varepsilon}{y} \tag{44}$$

From this expression and those developed for bending moment, moment-curvature or moment-strain curves can be developed. Figures 9 and 10 show plots of bending moment-curvature for a rectangular beam and tubular beam, respectively.

RESULTS AND CONCLUSIONS

Theoretical expressions for bending moment versus depth of elastic-plastic interface are developed for rectangular and circular tubes. These are specialized to give the moment for incipient yielding and the fully plastic condition. The expressions for the circular tubes are specialized to give results for a solid circular rod. The possibility of a strain singularity occurring when the elastic-plastic interface coincides with the NA is brought out. The assumption that "planes remain plane" is examined, and experimental data is presented which supports the assumption even when the plastic strains are of the same order as the elastic strains. The basis for determining residual or permanent strains and expressions for their calculation are given. Experimental data is given which shows that "planes remain plane" even for permanent strains resulting after unloading from elastic-plastic bending. A brief section discusses curvature and strain-curvature relationships.

From the results given we can see that the "planes remain plane" assumption used in elementary beam theory is also valid for deformations well into the elastic-plastic regime. Secondly, the rather simple theory presented can give some good insight into the complex area of elastic-plastic deformation by bending.

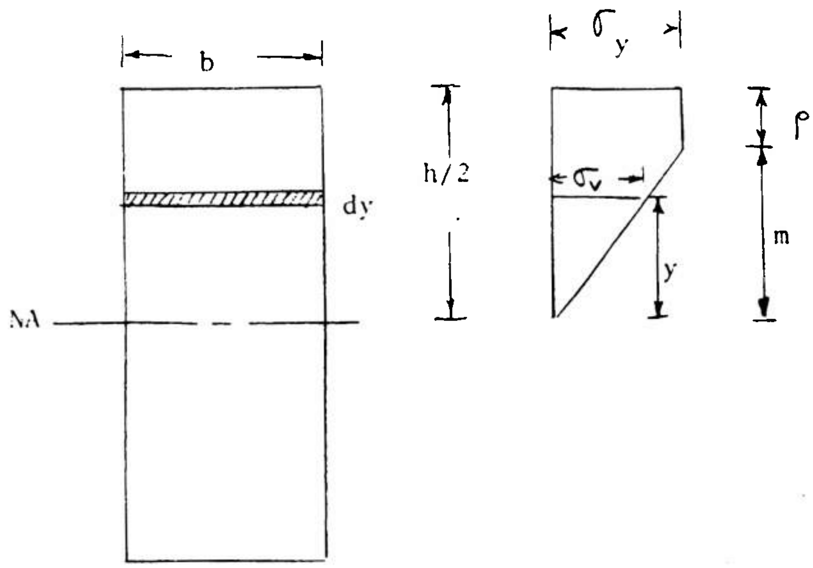


Figure 1. Cross section and stress distribution for rectangular beam.

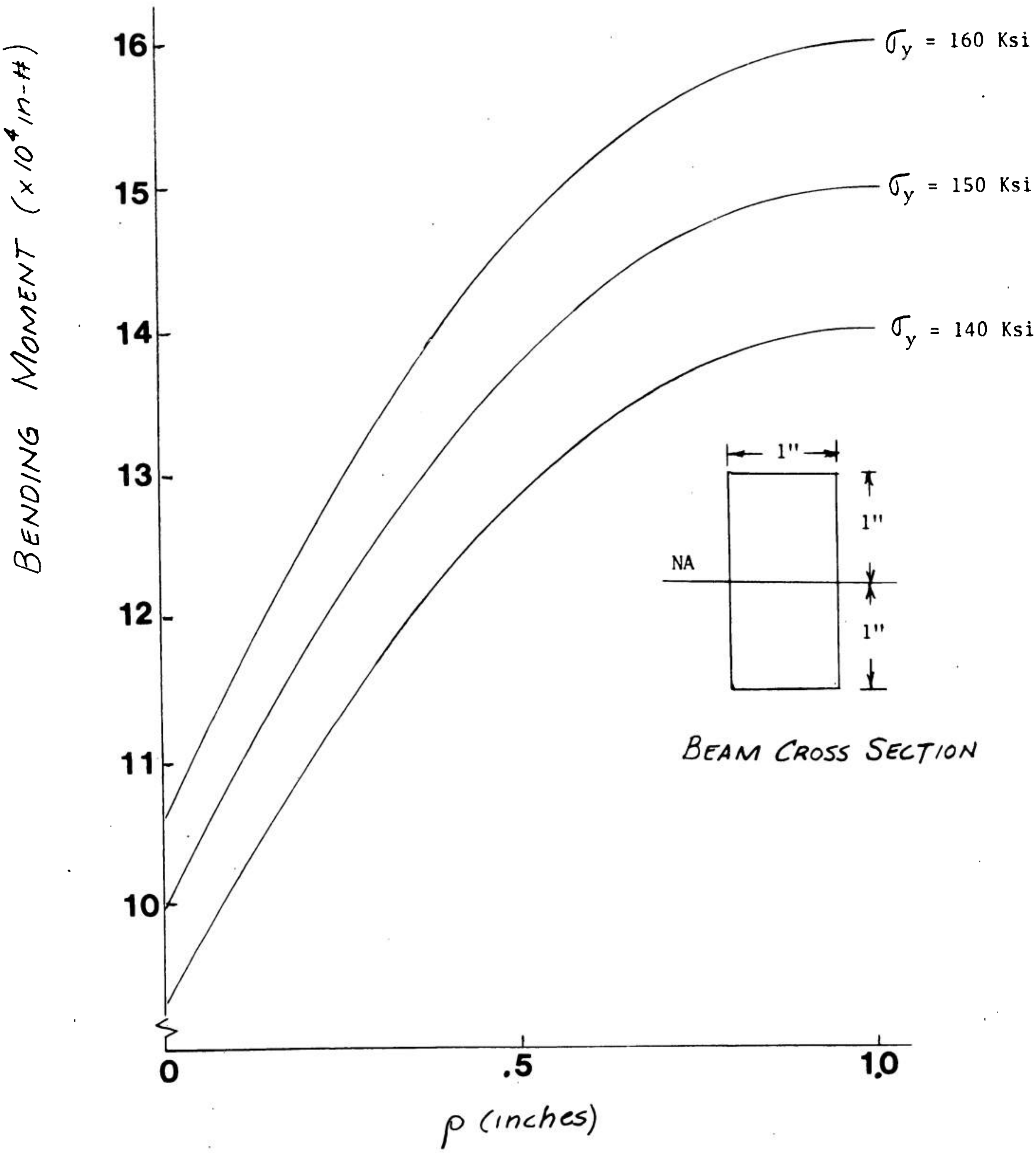
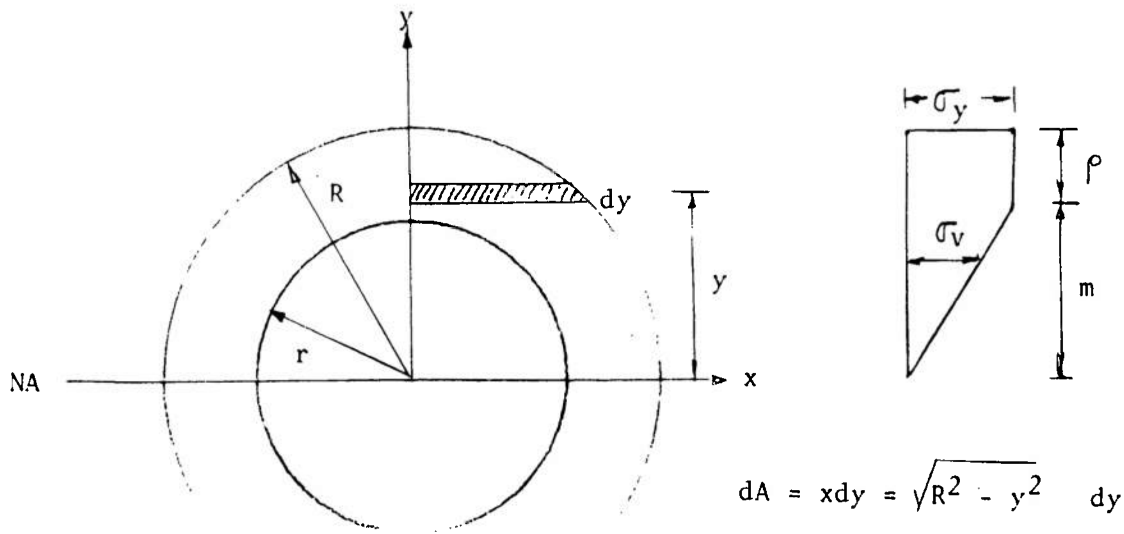
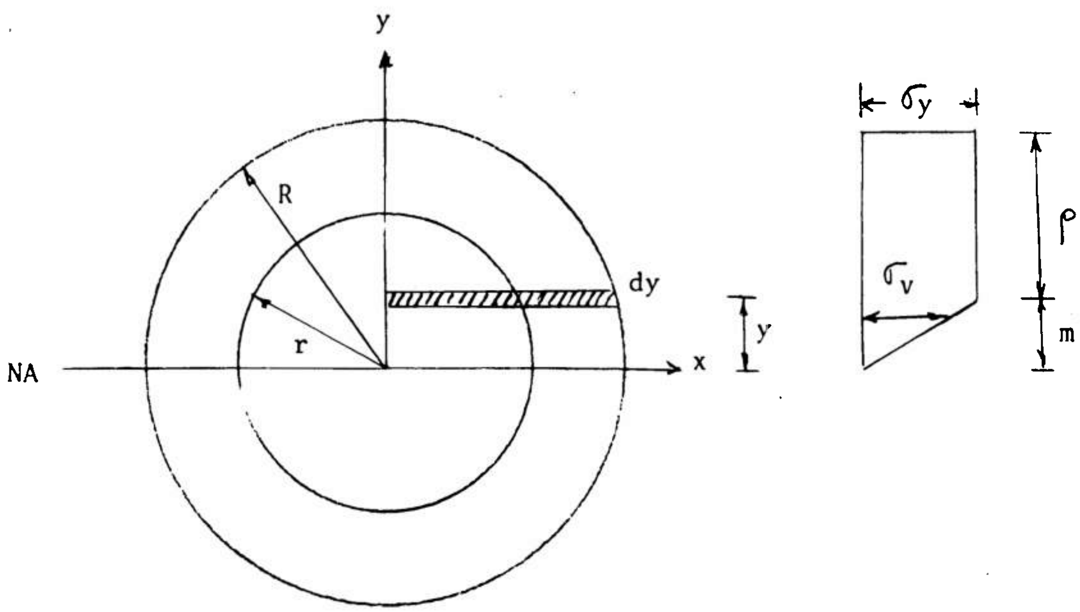


Figure 2. Bending moment vs. depth of elastic-plastic interface - rectangular beam.



$$dA = xdy = \sqrt{R^2 - y^2}\ dy$$

Figure 3. Cross section and stress distribution for circular tube - elastic-plastic interface between bore and outside fiber.



dA for large circle = $\sqrt{R^2 - y^2}$ dy

dA for small circle = $\sqrt{r^2 - y^2}$ dy

Figure 4. Cross section and stress distribution for circular tube - elastic-plastic interface between neutral axis and bore.

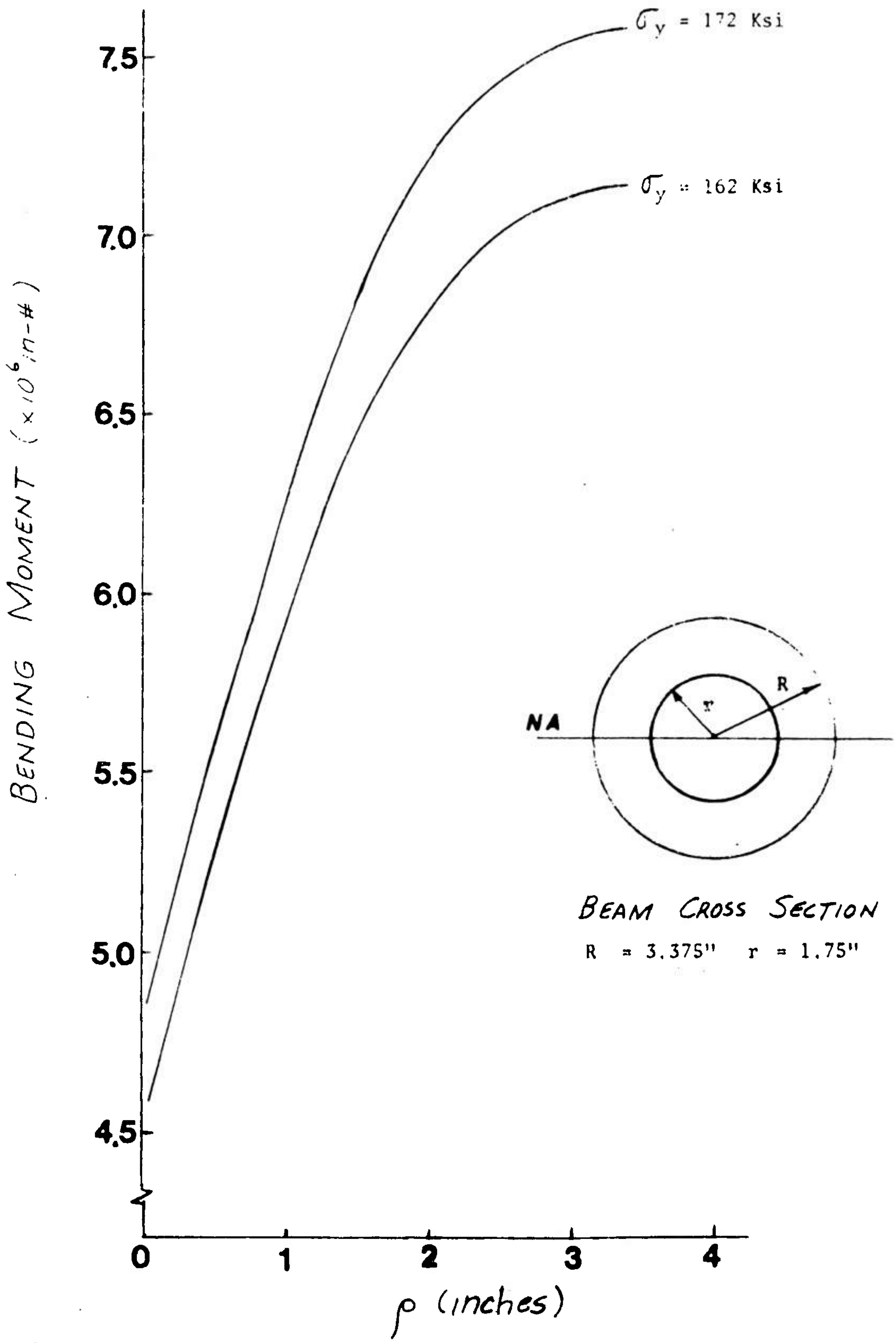


Figure 5. Bending moment vs. depth of elastic-plastic interface - circular tube.

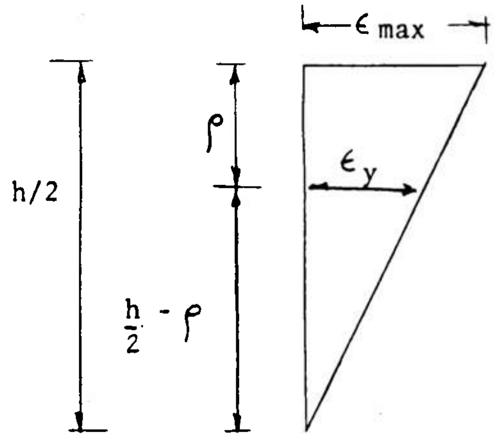


Figure 6. Strain distribution over the cross section for a rectangular beam.

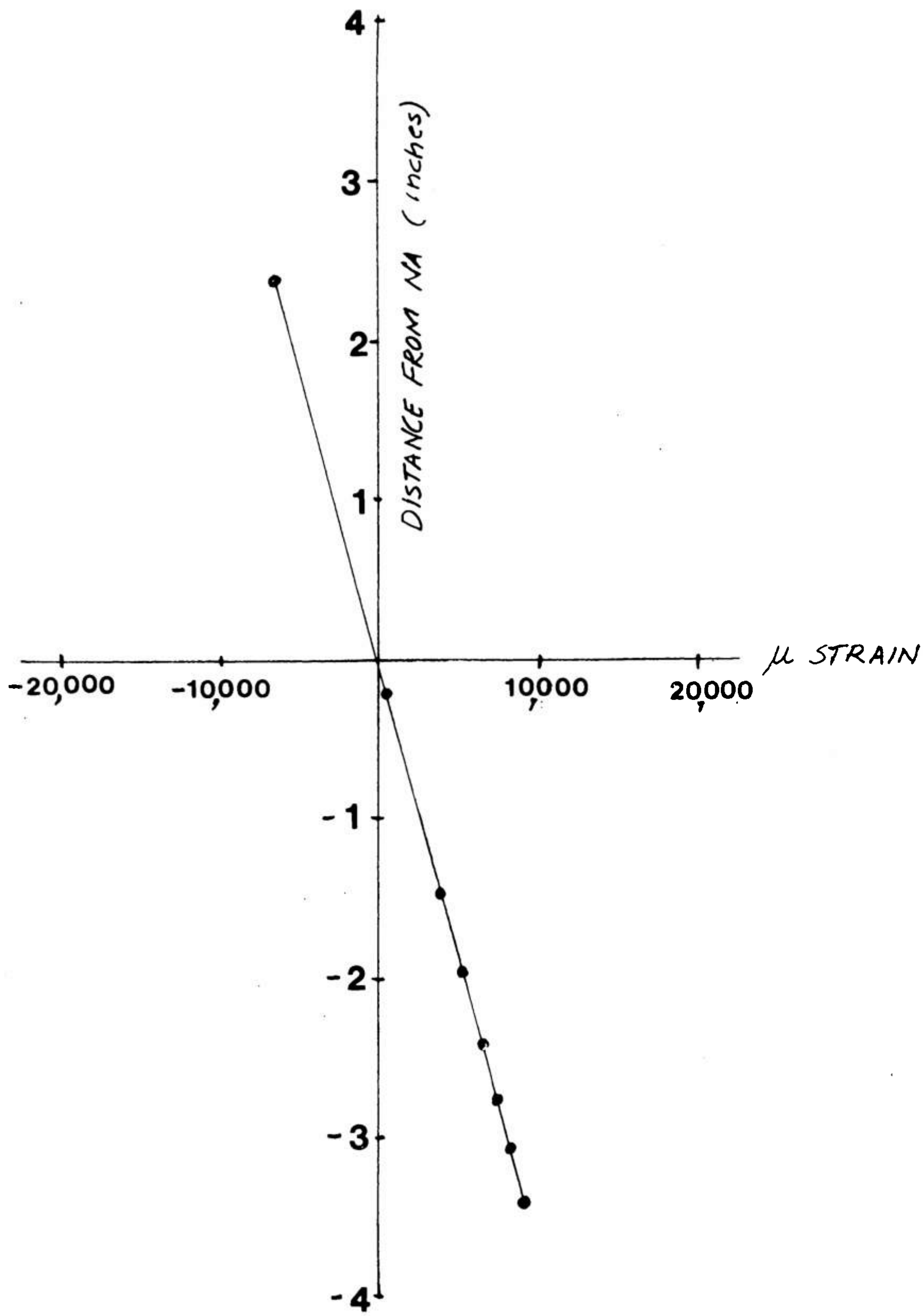


Figure 7. Experimental elastic-plastic strain vs. depth of cross section - circular tube.

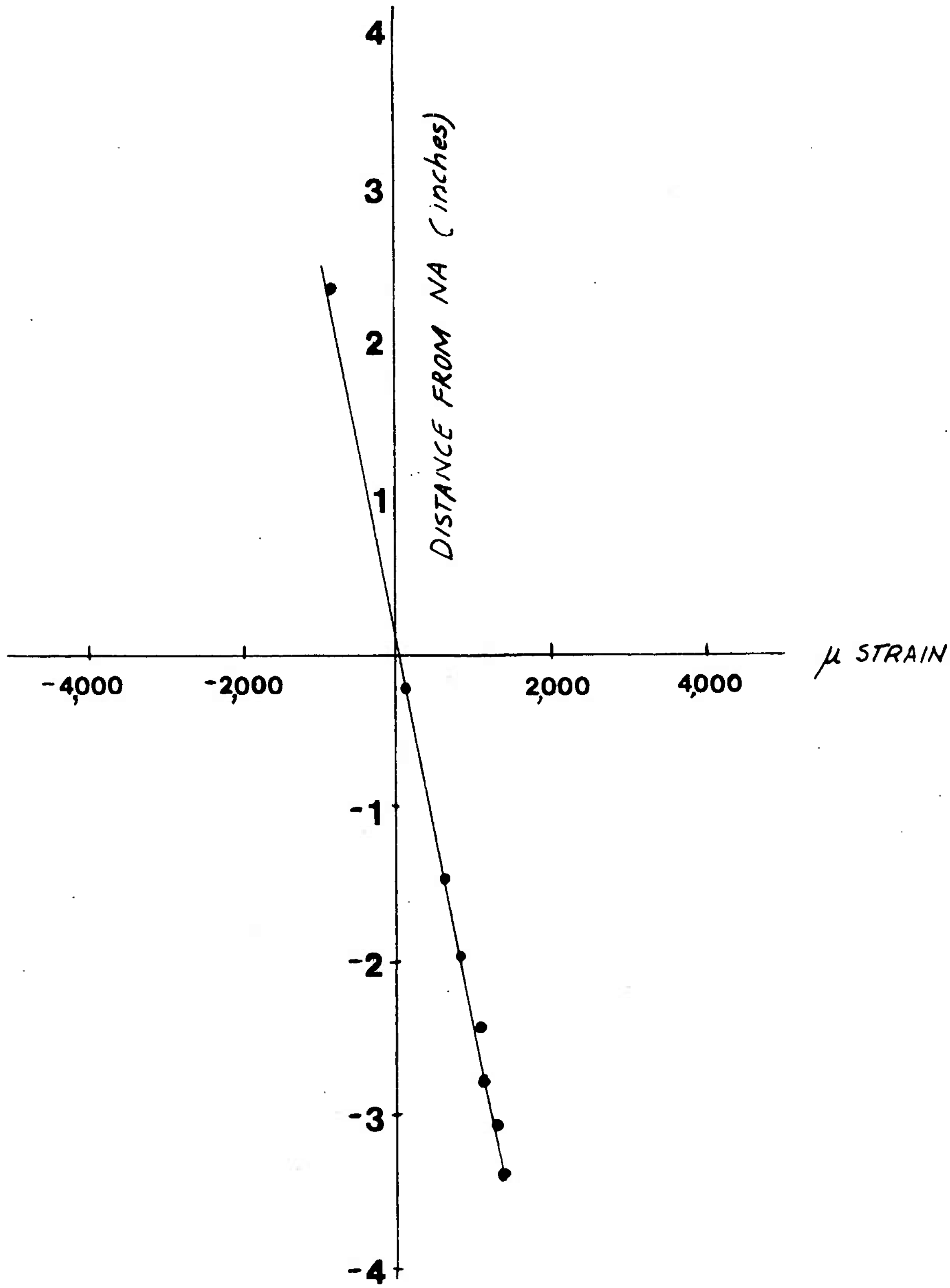


Figure 8. Experimental permanent strains vs. depth of cross section - circular tube.

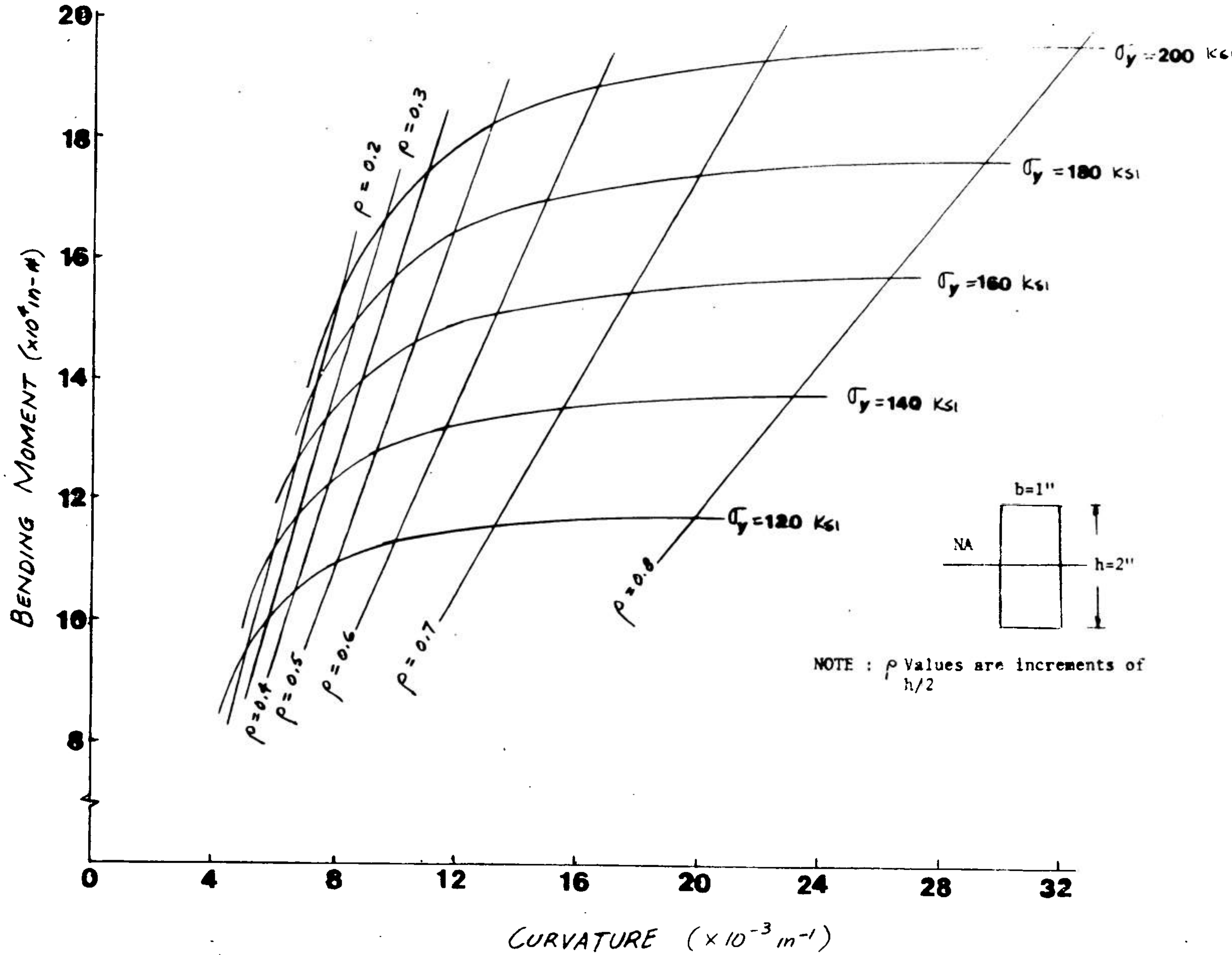


Figure 9. Bending moment vs. curvature for rectangular beam.

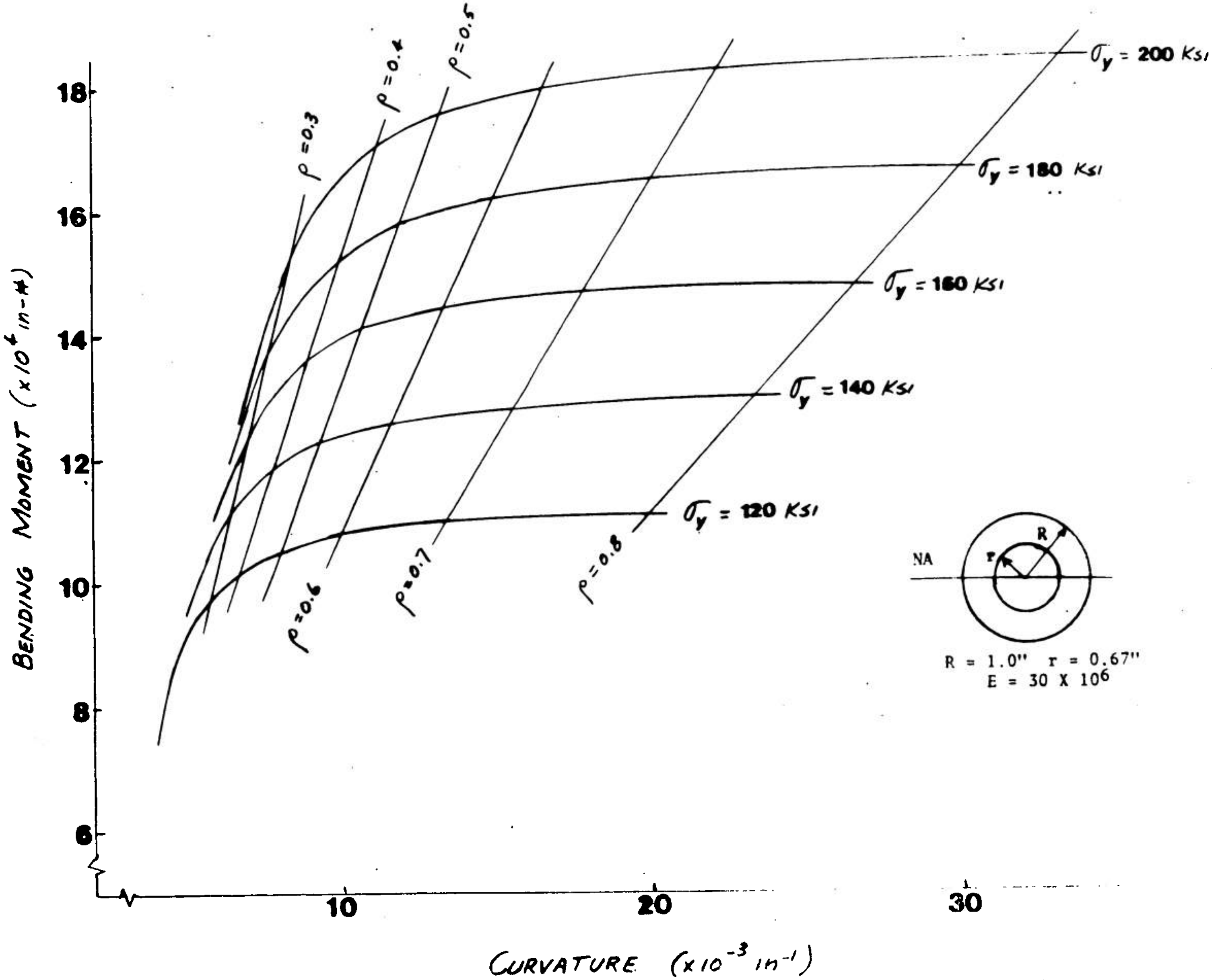


Figure 10. Bending moment vs. curvature for circular tube beam.

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COMMANDER HQ, US ARMY AVN SCH ATTN: OFC OF THE LIBRARIAN FT RUCKER, ALABAMA 36362	1
COMMANDER US ARMY FGN SCIENCE & TECH CEN ATTN: DRXST-SD 220 7TH STREET, N.E. CHARLOTTESVILLE, VA 22901	1
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NOTE: PLEASE NOTIFY COMMANDER, ARRADCOM, ATTN: BENET WEAPONS LABORATORY, DRDAR-LCB-TL, WATERVLIET ARSENAL, WATERVLIET, N.Y. 12189, OF ANY REQUIRED CHANGES.